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Leisure goods, education attainment and fertility choice

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Abstract This paper proposes a novel mechanism for the fertility decline that occurred across the world since the late nineteenth century. It suggests that the rise in the cost of children relative to leisure goods in the process of development contributed to the decline in fertility. The paper develops a unified growth model in which children are substitutes for leisure goods in the parental utility function. The theory suggests that the rise in income, the decline in the relative price of leisure goods and the increase in educational attainment in the process of development speed up the demographic transition from high to low fertility and contributed to the transition from stagnation to growth.

Keywords Fertility · Leisure goods · Technological progress · Growth

JEL Classification J13 · O11 · O33 · O40

1 Introduction

The demographic transition from high to low fertility that occurred across the world since the late nineteenth century is a by-product of economic development.[2] There are various theories in the literature that try to explain the reasons behind this phenomenon—for example, the decrease in mortality (e.g., Kögel and Prskawets 2001; Kalemli-Ozcan 2002, 2003; Lagerlof 2003; Doepke 2005; Soares 2005; Tamura 2006), the rise in income per capita (e.g., Becker 1981; Jones 2001), the old-age security hypothesis (e.g., Caldwell 1976; Boldrin and Jones 2002), the increase in educational attainment (e.g., Galor and Weil 1999; Galor and Weil

[1] The fertility transition began in developed countries towards the end of nineteenth century. The initial sharp decline in fertility was completed by World War II. After the baby boom, fertility has declined gradually ever since. In Asia and Latin America, the decline in population growth began towards the end of twentieth century. In African countries, the demographic transition is about to begin due to decreased rates of fertility. For more evidence, see Galor (2005).

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2000; Fernandez-Villaverde 2001; Greenwood and Seshadri 2002; Galor and Moav 2002; Doepke 2004) and the decline in the gender gap (e.g., Galor and Weil 1996).[3]

This paper presents a fundamentally different reason for raising children from the reasons that can be currently found in the literature. Hence it proposes a novel mechanism for the fertility decline. In the literature, children are treated as a type of a durable consumption good that provides a stream of satisfaction (or service) to parents over their lifetime. Substantial amount of such satisfaction is produced when parents engage in activities such as playing with and talking to their children. Francis and Ramey (2008) consider these as high enjoyment activities and classify them as leisure. In that sense, children can be considered as a leisure good which creates a leisure activity for their parents and hence a reason to have children. Under such circumstances, one could think of other conventional leisure activities as substitutes for the service provided by children. Interestingly, the consumption of such conventional leisure activities increases as economies develop.[4] We argue that such a trend in consumption behavior towards the conventional leisure activities may be an important driver behind the fertility decline observed over the course of economic development. The underlying mechanism depends on the observed increase in the price of children relative to the price of conventional leisure goods. In the literature, the biggest cost of raising children is time so that the real wage is often used as the price of children. As economies develop, the real wage increases while the price of leisure goods decreases relative to the general price level due to technological progress.[5] The hypothesis is then when leisure goods are relatively more expensive in the early stages of economic development (i.e., the ratio of real wages to the price of leisure goods is low), parents have many children to gain utility from leisure activities as the opportunity cost of children in terms of foregone consumption of the conventional leisure goods is low. Then the observed increase in the ratio of real wages to the price of leisure goods over time may induce parents to substitute leisure goods for children and hence fertility declines.

To expose the role of this mechanism theoretically, we extend Galor and Weil (2000) (henceforth “GW”) unified growth model. We choose GW’s model for the following crucial reason. Since Becker (1960), it has been a common practise in the economic analysis of fertility to consider that families derive utility from children (quantity augmented with or without quality) along with a single aggregate commodity (consumption) rather than the quantities of individual commodities. The reason for this is that, according to Becker (1960), there are no good or close substitutes for children. However, Becker admits that there may be many poor substitutes for children. GW assume a Cobb–Douglas utility function which implies no substitutes for children (quantity augmented with quality)—i.e., the elasticity of substitution between children and consumption is unity. We disaggregate the consumption set in GW into two broad categories—consumption good and leisure good, and consider a more general utility function in which the leisure good is a substitute for children for leisure activities while the consumption good is unrelated with those leisure activities.[6] Specifically,

[3] See Galor (2005) for an extensive review of the literature.

[4] See Sect. 2 for relevant evidence.

[5] See Sect. 2 for relevant evidence.

[6] Considering substitutes for children is not new in the literature. Jones (2001) shows that fertility declines as the negative effect of rising wages (the opportunity cost of children) dominates its positive income effect. For the parameter value generating this outcome, his utility function also implies that births and consumption are substitutes. However, his focus is on the property of the utility function rather than identifying potential substitutes for children—i.e., the whole consumption set is a substitute for children. In contrast, we identify leisure goods as substitutes for children from parents’ point of view and hence emphasize the role of a falling price of leisure goods on the fertility decline.

the leisure activities being a CES (Constant Elasticity of Substitution) function of children and the leisure good, are logarithmically separated from the consumption good. However, this specification implies that the consumption good is also a substitute for children and the leisure good under the parameter restriction that we consider.

The rest of the model is the following. All individuals are assumed to produce both goods using different technologies. They use the same technology in GW to produce the consumption good which combines land and effective units of labor. The leisure good is, on the other hand, manufactured from the consumption good and subject to endogenous technological change. This simple technology allows us to capture the decrease in the price of leisure goods. In the spirit of Becker (1965), agents in the economy produce leisure by combining the leisure good with leisure time. In addition, since we argue that children are a leisure good, time is also combined with children (quantity augmented with human capital of each child) to produce a leisure activity. We assume a Cobb–Douglas function for each leisure activity. Except for these modifications, the structure of the present model is the same as the GW model.

In this model, the decision rule for the optimal level of education (child quality) is the same as that in GW and depends positively on the rate of technological change expected to occur in the consumption good sector. The decision rule for the optimal number (quantity) of children, on the other hand, shows a hump-shaped relationship with the real wage, for a given level of education per child and a given price of the leisure good, rather than a positive (absent) relationship for low (high) income levels in GW. The reason can be explained by the conflict between the income and substitution effects. A rise in the real wage makes children relatively more expensive than the leisure and consumption goods and hence fertility tends to decrease. At the same time, it generates a positive income effect and hence the number of children tends to increase. Initially, at low income levels, the positive effect dominates and hence fertility increases. At sufficiently high income levels, however, the negative effect dominates so that fertility declines.[7] Moreover, an increase in education per child (the mechanism of GW for the fertility decline) and a decrease in the price of leisure good due to technological progress speed up this fertility transition.

The remainder of the paper is organized as follows. Section 2 discusses some evidence that supports the proposed mechanism. Section 3 discusses the model and solves it. Section 4 analyzes the evolution of the dynamical system of the model and conclusions are given in Sect. 5.

[7] This result is similar to that generated by the mechanism based on the rise in income in Becker (1981) and Jones (2001). This theory, however, appears to be inconsistent with the demographic transition in the Western Europe which differed significantly in terms of income per capita (e.g., Galor (2005). The feature that makes the current analysis different from these contributions is the decrease in the price of the leisure good. We do not find any studies documenting the relative price of leisure goods in these countries which would be useful to test the theory. However, one could argue that these countries and the US experienced a similar drop in the relative price of leisure goods during twentieth century as being the most advanced economies. In that sense, the new mechanism may have contributed to the fertility decline occurred at least during the twentieth century. Moreover, it is likely that the price of leisure goods declined in the nineteenth century. Although the Western European countries differed in income per capita, they experienced a similar growth rate of income per capita, indicating a similar rate of technological progress (e.g., Galor 2005). GW and Galor and Moav (2002) argue that the acceleration in technological progress led to an increase in the rate of return to investments in human capital in these countries. One could make a similar argument that the acceleration in technological progress also led to a decrease in the price of leisure goods. In addition, the nineteenth century was period of rapid growth for newspaper, magazines and books which was a dominant commercial recreation industry (e.g., Owen 1969). This can be attributed to the fertility decline observed before 1900, given the theory developed in this paper.

2 Supporting evidence

Malthus (1803) considers luxury as a potential check to population. He wrote “The discouragement to marriage, the consequent vicious habits, war, luxury, the silent though certain depopulation of large towns, and the close habitations, and insufficient food of many of the poor, prevent population from increasing beyond the means of subsistence;. . .”. He also wrote “It is a diffusion of luxury therefore among the mass of the people, and not an excess of it in a few, that seems to be most advantageous, both regard to national wealth and national happiness; . . . if, indeed, it be allowed that in every society, not in the state of a new colony, some powerful check to population must prevail; and if it be observed that a taste for comforts conveniences of life will prevent people from marrying, under the certainty of being deprived of these advantages; it must be allowed that we can hardly expect to find any check to marriage so little prejudicial to the happiness and virtue of society as the general prevalence of such a taste; and consequently, that the extension of luxury in this sense of the term is particularly desirable, and one of the best means of raising that standard of wretchedness alluded to in a former chapter.” Recent findings by Clark and Cummins (2010) may support Malthus’s view. According to them, fertility in England was higher for the rich in 1500–1780 but undifferentiable by class in 1780–1890. More importantly, they find that fertility for the rich started to decline during the Industrial Revolution (1760–1800) which was over 100 years before the classic demographic transition or around the time when Malthus was writing his essay.[8] The question is then why the demographic transition happened to the rich earlier. As viewed by Malthus, luxuries manufactured during the Industrial Revolution could have been a reason for this phenomenon. Recreational or other leisure goods are certainly luxuries in the early and intermediate stages of economic development as their consumption is limited to the wealthy few with sufficient income to purchase them. Then the diffusion of such luxury goods over time due to technological progress (in the form of increased income and reduced prices) among the middle and low income class families could explain the classic demographic transition as they were the bulk of the society. A piece of evidence based on the recreation expenditure elasticities supports this argument. The expenditure elasticity measures whether a good is a luxury (or how limited by income the consumption of a good is) with an expenditure elasticity greater than one indicating that the good is a luxury. Costa (1997) estimates the recreation expenditure elasticities in 1888–1890, 1917–1919, 1935–1936, 1972–1973 and 1991 using the US data. She finds that the expenditure elasticity decreased from around two to slightly more than one over the hundred years, implying that recreation became less of a luxury or more egalitarian. According to her, rising income, decreasing price of recreation and investment in public recreational goods are the main reasons behind the fall in the expenditure elasticity.[9]

The consumption of leisure goods increases as economies develop. According to Lebergott (1996), the expenditure share of leisure goods for an average American increased from 3% in 1900 to just over 8% in 2001. According to Kopecky (2005), if the set of leisure goods includes transportation goods and services for social and recreational activities, the expenditure share

[8] Although fertility declined for the rich, the overall fertility increased due to the increase in fertility for the lower income families which were the bulk of the society.

[9] See Owen (1969) for the development of the major commercial recreation industries such as sporting goods and equipment; phonographs, records, radios and television sets; motion pictures; newspapers, magazines and books; musical instruments between 1900 and 1961.

of leisure goods increased from 4% in 1900 to nearly 12% in 2001.[10] Owen (1969) shows that the demand for commercial recreation grew more rapidly within the 1909 and 1929 period (recreational expenditure as a percentage of total expenditures increased from 3.2 to 4.7%) than within the 1929 and 1961 period. Interestingly, fertility had fallen in the US until 1936. According to Mitchell (1998), births per 1000 population declined from 29.2 in 1909 to 17.6 in 1936. While the total expenditure share of leisure goods increased over time, its composition on different leisure products have changed as well. Costa (1997) shows the trends in the percentage of recreational expenditure on reading, movies and live entertainment, home entertainment and sporting equipments over the twentieth century. The share of reading decreased from 25.5% in 1888–1890 to 15.8% in 1991; the share of the second category initially increased from 8.7% in 1917–1919 to 23.8% in 1935–1936 before falling to 5.7% in 1991; the share of home entertainment increased continuously from 8.7% in 1917–1919 to 35.3% in 1991; the share of sporting equipment remained roughly the same around 8%.[11]

The theory developed in this paper argues that increases in the relative price of children to that of leisure goods are the main reason why fertility falls as economies enter the advanced stage of development. The following evidence shows that children have become a relatively more expensive good than leisure goods. As mentioned earlier, the real wage is considered to be the price of children in the literature. The relative price of children is then measured by the ratio of the real wage to the price of leisure goods. According to Williamson (1995), the real wage in the UK increased at an average annual rate of 0.88% between 1830 and 1913 and at an average of 1.32% between 1914 and 1945.[12] According to Kopecky (2005), the price of leisure goods relative to CPI declined by about 26% between 1900 and 1950. According to Owen (1969), the relative price of recreation goods decreased fast in the 1900 and 1929 period (recreation prices relative to cost of living indexes decreased by more than 20%), then slowed virtually no net change in the 1929 and 1961 period.

Owen (1969, 1971) show that the leisure time increased for non-student males in the 1900 and 1961 period which is explained by an increase in the real wage and a decrease in the relative price of recreation. In fact, leisure increased somewhat faster (about 10 hours a week) in the 1900 and 1929 period (when the relative price of recreation was falling faster) than that (7.5 hours a week) in the following 30 years (when the relative price of recreation remained roughly unchanged). In addition, according to Francis and Ramey (2008), leisure hours for American women aged between 25 and 54 also increased by 3.8 hours per week between 1900 and 1936. An increase in leisure time can be explained by the better and relatively cheaper leisure goods and hence can be attributed to the fertility decline observed in the US between 1900 and 1936.[13]

[10] Combining this with GDP per capita which increased by about a factor of seven over the same period implies that the spending on leisure goods increased by about a factor of 28. Data on GDP per capita and its growth rate for the US and other countries can be found in Maddison (2001).

[11] Owen (1969) points out that the nineteenth century was a period of rapid growth for newspaper, magazines and books which was a dominant commercial recreation industry. This can be attributed to the fertility decline observed in the nineteenth century. However, one can argue that leisure time spent on consuming these recreational goods replaces the time spent on other leisure goods such as chatting with other people. The theory developed in this paper suggest that people increased the consumption of leisure goods at the expense of reducing the time to play with or talk to children. The evidence provided here supports the theory. Francis and Ramey (2008) shows that leisure time increased slightly for all groups over the twentieth century. The decrease in fertility and the increase in the consumption of conventional leisure goods can explain this.

[12] See Williamson (1995) for data on real wages in other countries.

[13] One could argue that the increase in women's leisure time in the first half of the twentieth century is the result of improved and cheaper home appliances which increased the productivity of women in housework.

Cross-sectional data suggest that the rich have a fewer children than the poor in societies in the advanced stages of economic development (e.g., Jones and Tertilt 2008). According to the proposed mechanism, if everybody faces the same price of leisure goods, children are relatively more expensive for the rich than the poor in terms of forgone conventional leisure activities. The evidence provided by Jones and Tertilt (2008) suggest that the slope of the negative relationship between income and the number of children decreased substantially between 1828 and 1958. In modern societies, families tend to have more or less the same number of children irrespective of income levels. According to the proposed mechanism, conventional leisure goods that have become less of a luxury or more egalitarian could be the reason for this observed phenomenon.

Historical evidence shows that rural fertility is normally higher than urban fertility and the difference tends to decrease over time (see Fig. 1 in Greenwood and Seshadri 2002). According to Costa (1997), recreational expenditure elasticities could differ depending on the size of cities. The reason is that there could be differences in recreational opportunities between rural and urban areas. Rural areas and small cities do not have a sufficiently large population to support markets for many recreation opportunities on a permanent basis, implying that recreation can be more of a luxury in rural areas in some stages of development. This is consistent with the prediction of our mechanism for the fertility decline, that is, a higher relative price of leisure goods to children in rural areas leads to higher fertility than in urban areas. Over time, however, technological progress in the form of increased domestic recreational opportunities substituting market ones and increased income makes recreation more egalitarian. That is exactly what Costa (1997) finds—including demographic variables such as urbanization in the regressions mattered for the magnitude of the recreation expenditure elasticity in the early samples of 1988–1889 and 1917–1919 but no longer matters in the subsequent samples. The relevance of this evidence to the theory developed here is the observed decline in the difference between urban and rural fertility.

3 The model

We consider GW's overlapping-generations economy in which there are many identical individuals who live for two periods. As children in the first period of life, individuals are economically inactive and consume a fraction of their parents' time. As adults in the second period of life, they decide on the amount of consumption, the quantity (number) and quality (education) of their children, leisure and the labor market participation. A consumption set in GW is now disaggregated into two types of goods: consumption good, an index of goods that are unrelated with leisure activities and leisure good, an index of leisure goods that, to some extent, are substitutes for children for leisure activities. All adults produce the two goods using different technologies. The consumption good is produced using land and efficiency units of labor as in GW in which the land is exogenous and fixed over time and the quantity of efficiency units of labor is endogenously determined from households' optimization problem in the previous period. The production of the leisure good, on the other hand, uses a fraction

Footnote 13 continued
Although it was a possibility, one could also argue that diffusion of such home appliances could induce parents to choose more children by making it easier to raise children. Therefore, there must have been a greater opposite force induced parents to choose fewer children. This paper argues that the diffusion of leisure goods could have been the reason.

of the consumption good as an input. The demand for both goods together with the quantity and quality of children are determined by the households' decisions in each period.

3.1 Technology

Each adult produces z_t unit of the consumption good for each unit of time in period t in accordance with the following constant-returns-to-scale technology:

$$z_t = h_t^{\alpha} x_t^{1-\alpha} = h_t^{\alpha} \left(\frac{A_t X}{L_t} \right)^{1-\alpha} \tag{1}$$

where h_t is the amount of efficiency units of labor or human capital per adult, L_t is the size of the working age population, X is total (exogenous and constant) land, A_t is the level of land augmenting technology and $\alpha \in (0, 1)$ is the labor income share. The term $A_t X$ is total effective resources hence $x_t = A_t X / L_t$ is effective resources per adult.[14]

All working age individuals manufacture the leisure good by combining a fraction of the consumption good and the current state of technology in this sub-sector. Specifically, we adopt the following technology so as to simplify the analysis:

$$q_t = Z_t m_t, \tag{2}$$

where q_t is the units of output of the leisure good, Z_t is a productivity parameter, and m_t is the fraction of the consumption good. The maximization problem is given by:

$$\max_{m_t} \{ p_t Z_t m_t - m_t \}$$

where p_t is the unit price of the leisure good relative to that of the consumption good which is normalized to one. The optimal decision rule for this problem is $p_t = 1/Z_t$.

3.2 Preferences and budget constraints

The utility function of each adult member of generation t is assumed to be:

$$u = \gamma \log(c_t - \tilde{c}) + (1-\gamma) \log \left[\mu \left\{ (h_{t+1} n_t)^{\beta} l_t^{1-\beta} \right\}^{\sigma} + (1-\mu) \left\{ d_t^{\nu} u_t^{1-\nu} \right\}^{\sigma} \right]^{\frac{1}{\sigma}}. \tag{3}$$

According to (3), adults derive utility from the consumption good, c_t, in excess of its subsistence level, $\tilde{c} > 0$. In the literature, parents derive utility from children (child quantity augmented with or without quality of each child) *per se*. This paper proposes another way to look at this, that is, the service provided by children can be considered as a leisure activity for parents so that, in the spirit of Becker (1965), it requires both time and children. We assume that this leisure activity is a Cobb–Douglas function of children (child quantity, n_t, augmented with human capital of each child, h_{t+1}) and time input (leisure time with children), l_t.[15] Similarly, the leisure activity based on the conventional leisure good is a Cobb–Douglas function of the leisure good, d_t, and time devoted to consuming it (leisure time), u_t. In accordance

[14] Multiplying (1) by L_t yields the aggregate production function, $Y_t = H_t^{\alpha} (A_t X)^{1-\alpha}$ where H_t is the aggregate amount of efficiency units of labor—i.e., $H_t = h_t L_t$.

[15] GW consider potential income of each child, $w_{t+1} h_{t+1}$, rather than human capital per child, h_{t+1}, in the parental utility function. Since they consider a Cobb–Douglas utility function which yields the same results as the log-separated function, w_{t+1} does not play any role as individuals take it as given. In the current model, however, considering $w_{t+1} h_{t+1}$ would greatly complicate the analysis. For that reason, we follow Galor (2005) and consider only h_{t+1} in the utility function.

with Francis and Ramey (2008), it implies that the total leisure in this model is measured by $l_t + u_t$. An important feature of the utility function in (3) is that both leisure activities based on the leisure good and children are linked by a CES (Constant Elasticity of Substitution) function where μ and $1 - \mu$ denote their respective utility weights. The value of the parameter, σ, determines whether children and the leisure good are substitutes, complements or independent for leisure activities. The elasticity of substitution is then $1/(1 - \sigma)$. When $0 < \sigma \leq 1$ ($\sigma < 0$), they are substitutes (complements), implying that the marginal utility of children decreases (increases) with an increase in the amount of the leisure good. If $\sigma = 0$, the expression converges to a Cobb–Douglas function, implying that the marginal utility of children is independent of the leisure good because the underlying utility function is logarithmic. The following analysis considers the case with $0 < \sigma \leq 1$. Under such circumstances, the consumption good is also a substitute for children and the leisure good. However, the consumption good is neither a complement nor a substitute for the leisure activities—i.e., the elasticity of substitution between $c_t - \tilde{c}$ and $\left[\mu\left\{(h_{t+1}n_t)^{\beta} l_t^{1-\beta}\right\}^{\sigma} + (1-\mu)\left\{d_t^{\nu} u_t^{1-\nu}\right\}^{\sigma}\right]^{\frac{1}{\sigma}}$ is unity. Setting $d = 0$ and $\beta = 1$, we can derive the utility function considered by Galor (2005) which imply no substitutes for children in the economy as in GW. In that case, GW can be considered as a special case of the current model in which only children produce a leisure activity for their parents.

The adult is endowed with one unit of time which can be allocated between three mutually independent activities: working, childcare and leisure. Given her endowment of efficiency units of labor, h_t, the adult would earn potential income equal to z_t if she spent her entire time endowment in the labor market. Childcare requires only time as an input. Let τ^q be the fraction of adult's time associated with producing and raising one child regardless of quality and τ^e be the fraction of the adult's time connected with each level of education (quality) for each child, e_{t+1}. Since time devoted to raising children can be exchanged for the goods in the market, the opportunity cost of this activity is a fraction of her potential income: $(\tau^q n_t + \tau^e e_{t+1} n_t) z_t$ which can also be understood as the total spending on the purchase of children (both quality and quantity). In addition, leisure time that the adult enjoys with children and the leisure good can be exchanged for the consumption good in the market, its opportunity cost is a fraction of her potential income: $(l_t + u_t)z_t$. Her actual income is then determined by $(1 - \tau^q n_t - \tau^e e_{t+1} n_t - l_t - u_t)z_t$. Since the adult allocates her actual income between purchasing the consumption and leisure goods, her budget constraint is

$$c_t + p_t d_t = (1 - \tau^q n_t - \tau^e e_{t+1} n_t - l_t - u_t) z_t \tag{4}$$

where p_t is the relative price of the leisure good to that of the consumption good which is normalized to unity.

Unlike GW, we assume a constraint that governs the minimum number of children that parents want as in Jones (2001), that is,

$$n_t \geq \bar{n} > 0. \tag{5}$$

According to (5), the minimum number of children constraint binds if the optimal number of children derived from the optimization problem is smaller than it.[16]

Following GW, we assume that human capital of each child, h_{t+1}, depends on the expected rate of technological progress between periods t and $t + 1$ in the sector producing the

[16] The current model assumes that parents derive utility from children to gain satisfactions from a leisure activity. However, there could be other reasons why people have children such as old-age supports and intergenerational altruism. The minimum number of children constraint represents child quantity chosen by parents for those other reasons. Since we do not consider those reasons, the constraint is imposed exogenously.

consumption good, $g_{t+1} \equiv (A_{t+1} - A_t)/A_t$, and education per child, e_{t+1}, in an implicit fashion:

$$h_{t+1} \equiv h(g_{t+1}, e_{t+1}) \tag{6}$$

where $h(\cdot) > 0, h_g(\cdot) < 0, h_{gg}(\cdot) > 0, h_e(\cdot) > 0$, $h_{ee}(\cdot) < 0$, and $h_{eg}(\cdot) > 0 \, \forall (e_{t+1}, g_{t+1}) \geq 0$. The interpretation of these conditions is that the rate of technological progress has a negative effect on human capital and this "erosion effect" declines as g_{t+1} increases while education has a positive effect on human capital and its effect declines as e_{t+1} increases. The last property implies that technological progress increases the return to investments in education or education reduces the adverse effect of technological progress.[17]

3.3 Utility maximization

Given z_t, p_t and g_{t+1}, the adults choose $c_t, d_t, n_t, e_{t+1}, l_t$ and u_t to maximize their utility in (3) subject to the budget constraint in (4) and the minimum number of children constraint in (5). The first order conditions with respect to d_t, n_t, e_{t+1}, l_t and u_t are given by

$$\frac{\gamma p_t}{c_t - \tilde{c}} = \frac{\nu(1-\gamma)(1-\mu) d_t^{\nu\sigma-1} u_t^{(1-\nu)\sigma}}{\mu \left\{ (h_{t+1} n_t)^{\beta} l_t^{1-\beta} \right\}^{\sigma} + (1-\mu) \left\{ d_t^{\nu} u_t^{1-\nu} \right\}^{\sigma}}, \tag{7}$$

$$\frac{\gamma(\tau^q + \tau^e e_{t+1}) z_t}{c_t - \tilde{c}} = \frac{\beta(1-\gamma)\mu h_{t+1}^{\beta\sigma} n_t^{\beta\sigma-1} l_t^{(1-\beta)\sigma}}{\mu \left\{ (h_{t+1} n_t)^{\beta} l_t^{1-\beta} \right\}^{\sigma} + (1-\mu) \left\{ d_t^{\nu} u_t^{1-\nu} \right\}^{\sigma}}, \tag{8}$$

$$\frac{\gamma \tau^e n_t z_t}{c_t - \tilde{c}} = \frac{\beta(1-\gamma)\mu h_{t+1}^{\beta\sigma-1} n_t^{\beta\sigma} l_t^{(1-\beta)\sigma} h_e(\cdot)}{\mu \left\{ (h_{t+1} n_t)^{\beta} l_t^{1-\beta} \right\}^{\sigma} + (1-\mu) \left\{ d_t^{\nu} u_t^{1-\nu} \right\}^{\sigma}}, \tag{9}$$

$$\frac{\gamma z_t}{c_t - \tilde{c}} = \frac{(1-\beta)(1-\gamma)\mu h_{t+1}^{\beta\sigma} n_t^{\beta\sigma} l_t^{(1-\beta)\sigma-1}}{\mu \left\{ (h_{t+1} n_t)^{\beta} l_t^{1-\beta} \right\}^{\sigma} + (1-\mu) \left\{ d_t^{\nu} u_t^{1-\nu} \right\}^{\sigma}}, \tag{10}$$

$$\frac{\gamma z_t}{c_t - \tilde{c}} = \frac{(1-\nu)(1-\gamma)(1-\mu) d_t^{\nu\sigma} u_t^{(1-\nu)\sigma-1}}{\mu \left\{ (h_{t+1} n_t)^{\beta} l_t^{1-\beta} \right\}^{\sigma} + (1-\mu) \left\{ d_t^{\nu} u_t^{1-\nu} \right\}^{\sigma}}. \tag{11}$$

In (7), the expression on the left-hand side represents the utility cost generated from purchasing one unit of the leisure good measured by the forgone consumption good while the utility gain is on the right-hand side. In (8), the expression on the right-hand side shows the utility gain of having one child while its utility cost is on the left-hand side as it decreases the amount of the consumption good by decreasing the adult's labor market participation. The price the adult pays for each child quantity is $(\tau + \tau^e e_{t+1}) z_t$ which is increasing in z_t as well as in e_{t+1} as the same level of education has to apply to each child. The expression in (9) shows the utility gain of purchasing an extra unit of education for each child on the right-hand side while the utility cost is on the left-hand side which is measured by the forgone consumption good through a decrease in the adult's time available for generating income. The price paid for the extra unit of education is $\tau^e n_t z_t$ which is an increasing function of

[17] Lagerlof (2006) considers an explicit functional form for $h(g_{t+1}, e_{t+1})$, that is $h_{t+1} = \frac{e_{t+1}+\rho\tau}{e_{t+1}+\rho\tau+g_{t+1}}$ where $\rho \in (0, 1)$ is an exogenous part of the total fixed time cost of raising children, τ, that contributes towards building human capital—i.e., children acquire some knowledge while being raised (public good) which is, however, not as effective as formal education. Therefore $e_{t+1} + \rho\tau$ is effective education.

z_t as well as of n_t because an additional unit of education must apply to more units. The presence of n_t in the price of quality and that of e_{t+1} in the price of quantity will ensure the classic interaction between the quality and quantity of children when the other things such as parental income change (e.g., Becker and Lewis 1973). The expressions in (10) and (11) are the usual labor-leisure tradeoff.

We divide (8) by (9) and obtain the same expression as in GW:

$$G(e_{t+1}, g_{t+1}) \begin{cases} \leq 0 & \text{if } e_{t+1} = 0 \\ = 0 & \text{if } e_{t+1} > 0 \end{cases} \tag{12}$$

where $G(e_{t+1}, g_{t+1}) = (\tau^q + \tau^e e_{t+1}) h_e(g_{t+1}, e_{t+1}) - \tau^e h(e_{t+1}, g_{t+1})$. Assuming $G(0, 0) < 0$, GW derive the following decision rule for the optimal level of education per child:

$$e_{t+1} = e(g_{t+1}) \begin{cases} = 0 & \text{if } g_{t+1} \leq \hat{g} \\ > 0 & \text{if } g_{t+1} > \hat{g} \end{cases} \tag{13}$$

where $\hat{g} > 0$ and $e'(g_{t+1}) > 0$ for any $g_{t+1} > \hat{g}$. According to (13), the optimal level of education per child is zero when the rate of technological progress is sufficiently low, but positive and increases with g_{t+1} for sufficiently fast technological progress. According to GW, a decrease in the level of human capital due to the erosion effect of technological progress is reduced by an increase in education for $g_{t+1} > \hat{g}$. The implication is that the overall effect of technological progress on human capital is still negative—i.e., $h_g\,(g_{t+1}, e(g_{t+1})) < 0$.[18] This property is reflected in Lagerlof's (2006) choice of functional form for $h(g_{t+1}, e_{t+1})$. The optimal level of education per child hence their human capital are, however, independent of parental potential income, z_t. The reason can be explained using (8) and (9). Other things being equal, an increase in z_t tends to increase the demand for both e_{t+1} and n_t by decreasing the marginal utility of the consumption good, $(1-\gamma)/(c_t - \tilde{c})$, implying that they are normal goods—i.e., it generates the wealth effect. At the same time, it produces a substitution effect on the demand for both commodities by increasing their prices $\tau^e n_t z_t$ and $(\tau^q + \tau^e e_{t+1}) z_t$: directly as well as indirectly through n_t for the former and through e_{t+1} for the latter. This interaction between the quality and quantity of children leaves the level of education per child unaffected to the changes in z_t. In other words, the income and substitution effects are cancelled out.

After some manipulations, one may obtain the following expressions for the optimal quantities of children, the leisure good and leisure:

$$n_t = \max\left\{\bar{n}, \frac{(1-\gamma)\beta\,(1-\tilde{c}/z_t)}{(\tau^q + \tau^e e_{t+1})(1+\Omega_t)} \equiv n(p_t, g_{t+1}, z_t)\right\}, \tag{14}$$

$$d_t = N\left(\frac{z_t^{1-(1-\nu)\sigma}(\tau^q + \tau^e e_{t+1})^{1-(1-\beta)\sigma}}{p_t^{1-(1-\nu)\sigma} h_{t+1}^{\beta\sigma}}\right)^{\frac{1}{1-\sigma}} n_t \equiv d(p_t, g_{t+1}, z_t) \tag{15}$$

$$l_t = B(\tau^q + \tau^e e_{t+1}) n_t \equiv l(p_t, g_{t+1}, z_t) \tag{16}$$

$$u_t = D\frac{p_t}{z_t} d_t \equiv u(p_t, g_{t+1}, z_t) \tag{17}$$

[18] This assumption simplifies the following analysis greatly. In the other cases, an increase in education would either fully recover or more than fully recover the loss in human capital—i.e., $h_g\left(e(g_{t+1}), g_{t+1}\right) \geq 0$. While the former would not affect the qualitative results, the latter would lead to more general results.

where $N = \left(\frac{(1-\mu)\nu D^{(1-\nu)\sigma}}{\mu\beta B^{(1-\beta)\sigma}}\right)^{\frac{1}{1-\sigma}}$, $D = \frac{1-\nu}{\nu}$, $B = \frac{1-\beta}{\beta}$, $\Omega_t = M\left(\frac{z_t^{\nu}(\tau^q+\tau^e e_{t+1})^{\beta}}{h_{t+1}^{\beta} p_t^{\nu}}\right)^{\frac{\sigma}{1-\sigma}}$ and $M = \left(\frac{(1-\mu)\nu^{\sigma} D^{(1-\nu)\sigma}}{\mu\beta^{\sigma} B^{(1-\beta)\sigma}}\right)^{\frac{1}{1-\sigma}}$. Let $n(\cdot), d(\cdot)$, $l(\cdot)$ and $u(\cdot)$ be shorthand notations for $n(p_t, g_{t+1}, z_t), d(p_t, g_{t+1}, z_t), l(p_t, g_{t+1}, z_t)$ and $u(p_t, g_{t+1}, z_t)$ respectively where we use $e_{t+1} = e(g_{t+1})$ and $h_{t+1} = h(e(g_{t+1}), g_{t+1}) = h(g_{t+1})$. The functional properties of $n(\cdot), d(\cdot), l(\cdot)$ and $u(\cdot)$ are summarized in the following proposition.[19]

Proposition 1 *Other things being equal:*

(a) *a decrease in the price of the leisure good, p_t, leads to a decrease in both n_t and l_t, but an increase in both d_t and u_t —i.e.,*

$$n_p(\cdot) > 0, l_p(\cdot) > 0, \quad d_p(\cdot) < 0 \text{ and } u_t(\cdot) < 0,$$

(b) *technological progress expected to occur between time t and $t+1$ in the sector producing the consumption good, g_{t+1}, has a negative effect on both n_t and l_t but a positive effect on both d_t and u_t for all g_{t+1}—i.e.,*

$$n_g(\cdot) < 0, \quad l_g(\cdot) < 0, \text{ and } d_g(\cdot) > 0 \text{ and } u_g(\cdot) > 0,$$

(c) *there exists a time varying critical value $\tilde{z}_t \equiv \tilde{z}(p_t, g_{t+1})$ so that an increase in the parental potential income, z_t, has (c.1) a positive effect on n_t, d_t, l_t and u_t if $z_t < \tilde{z}_t$ and (c.2) a negative effect on both n_t and l_t but a positive effect on both d_t and u_t if $z_t > \tilde{z}_t$—i.e.,*

$$n_z(\cdot) \geq 0, \quad l_z(\cdot) \geq 0, \quad d_z(\cdot) > 0 \text{ and } u_z(\cdot) > 0 \text{ if } z_t \leq \tilde{z}_t, \tag{c.1}$$

$$n_z(\cdot) \leq 0, \quad l_z(\cdot) \leq 0, \quad d_z(\cdot) > 0 \text{ and } u_z(\cdot) > 0 \text{ if } z_t \geq \tilde{z}_t. \tag{c.2}$$

where $\tilde{z}_t > \frac{\tilde{c}(1-\sigma(1-\nu))}{\sigma\nu} > \tilde{c}$ for $\sigma \in (0, 1)$. In addition, $\tilde{z}_g(\cdot) < 0$ and $\tilde{z}_p(\cdot) > 0$.

Proof See Appendix. □

The intuitions of the results in Proposition 1 are the following. The relative price of child quantity to that of the leisure good is reflected in Ω_t: an increase in both z_t and e_{t+1} but a decrease in p_t cause an increase in Ω_t. The results in part (a) can be explained on the basis of the substitutability between children and the leisure good: the lower the price of the leisure good, the lower the demand for children (hence less leisure time with children) but the higher the demand for the leisure good (hence more corresponding leisure time). Consequently, the effect of cheaper leisure goods on the overall leisure, $l_t + u_t$, is ambiguous.

An increase in g_{t+1} has three negative effects on n_t. GW's quality-quantity tradeoff is one of them which makes n_t more expensive relative to e_{t+1} for $g_{t+1} > \hat{g}$. In general, even a small initial increase in e_{t+1} due to an increase in g_{t+1} can lead to a large decrease (increase) in n_t (e_{t+1}) if the interaction between quality and quantity is strong (e.g., Becker and Lewis 1973). The second negative effect makes child quantity more expensive relative to the leisure good by increasing Ω_t for $g_{t+1} > \hat{g}$. In that sense, the current setup strengthens the usual interaction between e_{t+1} and n_t. If there are no substitutes for children—i.e., σ converges to zero, this effect will diminish to zero. The last effect of g_{t+1} on n_t arises through a decrease in h_{t+1}. As parents derive utility from the level of human capital of each child, a decrease in h_{t+1} decreases the marginal utility of child quantity according to (8) for $\sigma \in (0, 1)$. Again,

[19] The equilibrium conditions of the economy are as follows: $d_t = q_t$ (the leisure good sector) and $c_t + p_t d_t = c_t + m_t = (1 - \tau^q n_t - \tau^e e_{t+1} n_t - l_t - u_t) z_t$ (the consumption good sector) as $q_t = Z_t m_t$ and $p_t = 1/Z_t$.

if there are no substitutes for children ($\sigma = 0$), this effect will vanish. According to (16), an increase in education due to an increase in g_{t+1} tends to increase l_t. However this effect is dominated by its negative effect through n_t. The increase in g_{t+1}, on the other hand, has a positive effect on d_t and u_t as its positive effect through e_t and h_{t+1} dominates its negative effect through n_t.[20] Again, the effect of accelerating technological progress on the overall leisure, $l_t + u_t$, is ambiguous.

An increase in z_t has opposite effects on both n_t and l_t: the income (positive) and substitution (negative) effects. In a simple model with $\tilde{c} = 0$ and $\sigma = 0$, these effects are exactly offset, leaving n_t and l_t unchanged. Introducing $\tilde{c} > 0$ strengthens the income effect hence both n_t and l_t increase. The strength of this additional income effect, however, becomes weaker as z_t increases. In the current setup with $\sigma > 0$, the substitution effect is also stronger than usual due to $\Omega_t > 0$. At $\tilde{z}_t$, the sum of the usual and additional income effects is exactly offset by the stronger substitution effect. The case with $z_t < \tilde{z}_t$ implies that the positive effects dominate the substitution effect hence the number of children increases with z_t. In other words, children are relatively cheaper than the consumption and leisure goods in this region. In the case with $z_t > \tilde{z}_t$, however, children are relatively more expensive than the goods (or the substitution effect dominates the income effect) hence child quantity decreases with z_t. In other words, the model generates a hump-shaped relation between n_t and z_t for given p_t and g_{t+1}. A decrease in p_t and an increase in g_{t+1} make the transition from $n_z(\cdot) > 0$ to $n_z(\cdot) < 0$ faster by decreasing $\tilde{z}_t$. Leisure time with children follows the pattern of the optimal number of children. The optimal amount of the leisure good increases with z_t for two reasons. Firstly, the leisure good is a normal good. Secondly, it is a substitute for children whose price increases with z_t, leading to an increase in Ω_t. Although an increase in z_t has a direct negative effect on u_t as being its opportunity cost, it is dominated by its indirect positive effect through d_t. Given the properties of l_t and u_t, the effect of increasing income on the overall leisure, $l_t + u_t$, is positive for $z_t \leq \tilde{z}_t$, but is ambiguous for $z_t > \tilde{z}_t$.[21]

Using the results in Proposition 1 and the subsequent discussions, one can clearly differentiate the new mechanism from GW's quality-quantity tradeoff mechanism in terms of their effects on fertility by fixing g_{t+1} at a constant rate. Consequently, e_{t+1} and h_{t+1} are constant while n_t, d_t, l_t and u_t are determined by only z_t and p_t. In this case, it is straightforward to see that the results in sections (a) and (c) of Proposition 1 are still valid. For example, the model still generates the hump-shaped relationship between fertility and income and decreases in the price of the leisure good make the transition faster from the pre-Modern Growth regime where fertility is positively related with income to the Modern Growth regime in which the relationship is switched into negative.

[20] For the functional forms used by Lagerlof (2006), we find that both d_t and u_t increase with g_{t+1}.

[21] The positive relationship between the overall leisure and income for $z_t \leq \tilde{z}_t$ may be inconsistent with the historical evidence observed in England where leisure time fell initially and picked up after the onset of the Industrial Revolution (e.g., Voth 1998). An alternative way of modelling l_t and u_t (that is, they do not appear in the utility function but only in the time constraint as $l_t = \theta n_t$ and $u_t = \vartheta d_t$ where θ and ϑ are time cost of enjoying one child and one unit of the leisure good respectively) while maintaining the current structure of the utility function does not change this result. It requires additional assumptions to bring the model to fit this evidence. For example, one could assume that d_t remains constant in the early stages of development and the efficiency of consuming the leisure good increases (i.e., ϑ falls) such that u_t falls. This would provide a case where the fall in u_t may dominate the rise in l_t. This is a simple extension which maintains the direct effects of p_t and z_t on u_t in (17) but minimizes the indirect effects through d_t.

3.4 Technological progress

The rate of technological progress that occurs between time t and time $t+1$ in the sector producing the consumption good, g_{t+1}, is the same as that in GW, that is, an implicit function of education at time t, e_t, and the size of working age population at time t, L_t:

$$g_{t+1} \equiv \frac{A_{t+1} - A_t}{A_t} = g(e_t, L_t) \tag{18}$$

where for $L_t \gg 0$, $g(0, L_t) > 0$, $g_i(\cdot) > 0$, and $g_{ii}(\cdot) < 0, i = e_t, L_t$. The rate of technological progress is an increasing concave function of each determinant for a sufficiently large population size. Moreover, there is positive technological progress even if education is zero.[22] GW assume $g_L(0, L_t) = 0$ for a sufficiently small population size to ensure that early stages of development take place in a Malthusian steady state. This assumption is kept in our analysis.

We assume that the rate technological change in the production of the leisure good between time t and time $t+1$, g^Z_{t+1}, is also determined by e_t and L_t at time t. Specifically,

$$g^Z_{t+1} \equiv \frac{Z_{t+1} - Z_t}{Z_t} = g^Z(e_t, L_t) \tag{19}$$

where $L_t \gg 0$, $g^Z(0, L_t) > 0$, $g^Z_i(\cdot) > 0$, and $g^Z_{ii}(\cdot) < 0, i = e_t, L_t$. In addition, we assume $g^Z(0, L_t) = 0$ for a sufficiently small size of population to ensure that early stages of development is indeed characterized by a Malthusian steady state. In the following analysis, the dynamical system of the economy will be initially studied under this assumption. We will then analyze the effect of $g^Z_{t+1} > 0$ on the evolution of the dynamical system.

3.5 Population, technology and effective resources

The evolution of the size of working population, L_t, technology, A_t , and effective resources per worker, x_t, is the same as those in GW and is governed by the following three difference equations:

$$L_{t+1} = n_t L_t, \tag{20}$$

$$A_{t+1} = (1 + g_{t+1})A_t, \tag{21}$$

$$x_{t+1} = \frac{1 + g_{t+1}}{n_t} x_t \tag{22}$$

where their initial levels are historically given at L_0, A_0 and $x_0 = (A_0 X)/L_0$ respectively. The number of children per person, n_t, and the rate of technological progress, g_{t+1}, are determined by the expressions in (14) and (18) respectively.

Using (19), the evolution of the relative price of the leisure good, $p_t = 1/Z_t$, can be written as

$$p_{t+1} = \frac{1}{1 + g^Z_{t+1}} p_t \tag{23}$$

where its initial value, p_0, is historically given. $g^Z_{t+1} > 0$ implies that the leisure good becomes more affordable over time.

[22] Lagerlof (2006) considers the explicit form for $g(e_t, L_t)$, that is, $g_{t+1} = (e_t + \rho\tau) \min\{\theta L_t, a^*\}$ where $\theta > 0$ measures the "scale" effect of L_t while $a^* > 0$ corresponds to $\lim_{L\to\infty} g(\cdot) = a^*$ for given e_t. Thus population increases technological progress linearly for $L_t \le a^*/\theta$ and then has no effect.

4 The dynamical system

This section analyzes the dynamical system of the economy which determines its development through the evolution of population, income per capita, technology levels in the production of both the consumption and leisure goods, education per worker, human capital per worker and effective resources per worker. The sequence that determines the development of the economy in GW, $\{e_t, g_t, x_t, L_t\}_{t=0}^{\infty}$, is now extended to $\{e_t, g_t, x_t, L_t, p_t\}_{t=0}^{\infty}$ in the current analysis that satisfies (18–23).

Since we do not follow GW in solving for the household's optimization problem, the dynamical system is characterized by one regime rather than two.[23] For a given size of population L, and a given price of the leisure good p, the development of the economy is determined by the following three-dimensional nonlinear system of difference equations:

$$\begin{cases} e_{t+1} = e(g(e_t); L) \\ g_{t+1} = g(e_t; L) \\ x_{t+1} = \phi(e_t, g_t, x_t; L, p)x_t \end{cases} \tag{24}$$

where $\phi(e_t, g_t, x_t; L, p) \equiv (1+g_{t+1})/n_t$ and the initial values e_0, g_0 and x_0 are historically given.

The evolution of e_t and g_t is independent of x_t. Therefore, the analysis of the joint dynamics of education and technology is exactly the same as that in GW. Broadly speaking, this dynamical subsystem is characterized by three different configurations in the (e_t, g_t) space, depending on the size of population. For a small population size, there is a unique globally stable steady-state equilibrium $(\bar{e}, \bar{g}) = (0, g^l)$ characterizing the dynamical subsystem (see Fig. 1). For a moderate population size, the dynamical subsystem is characterized by multiple steady-state equilibria: $(\bar{e}, \bar{g}) = (e^u, g^u)$ is unstable and lies between $(\bar{e}, \bar{g}) = (0, g^l)$ and $(\bar{e}, \bar{g}) = (e^h, g^h)$ which are stable. This is depicted in Fig. 2. Figure 3 shows the dynamical subsystem for a large population size which is characterized by a globally stable steady-state equilibrium $(\bar{e}, \bar{g}) = (e^h, g^h)$.

4.1 Global dynamics

We analyze the evolution of the dynamical system of the economy in (24) using a series of phase diagrams in the (e_t, x_t) space, as described in GW. Each phase diagram, shown in Figs. 4, 5 and 6, has three components: the Malthusian Frontier which separates one regime where parental potential income has a positive effect on the chosen number of children from the other where the effect is reversed, the XX locus along which the effective resources per worker is constant and the EE locus along which the level of education per worker is constant. There is one similarity and two differences in the phase diagrams between ours

[23] The utility maximization approach in GW is slightly different from the one here. They impose the subsistence consumption constraint externally which generates corner solutions whereas we maximize a Stone-Geary utility function in which the subsistence consumption constraint is incorporated. Thus n_t in GW increases with z_t for $z_t < \tilde{z}$, but remains constant for $z_t > \tilde{z}$. If they assumed a Stone-Geary utility function, the decision rule for n_t would be $n_t = \frac{\gamma(1-\tilde{c}/z_t)}{\tau^q+\tau^e e_{t+1}}$ according to which n_t increases and converges to $\frac{\gamma}{\tau^q+\tau^e e_{t+1}}$ smoothly as z_t converges to infinity for given e_{t+1}. The decision rule in GW is qualitatively similar to this rule in the sense that the same convergence is not smooth and takes place immediately when $z_t = \tilde{z}$. The benefit of their approach is that the dynamical system is divided into two regions by $\tilde{z}$. Otherwise, the entire space (x_t, e_t), in which the system is analyzed, would be the Malthusian region where z_t has always a positive effect on n_t as there is no $\tilde{z}$. As can be seen from Proposition 1, the current model generates $\tilde{z}_t$ without imposing the subsistence consumption constraint externally as in GW.

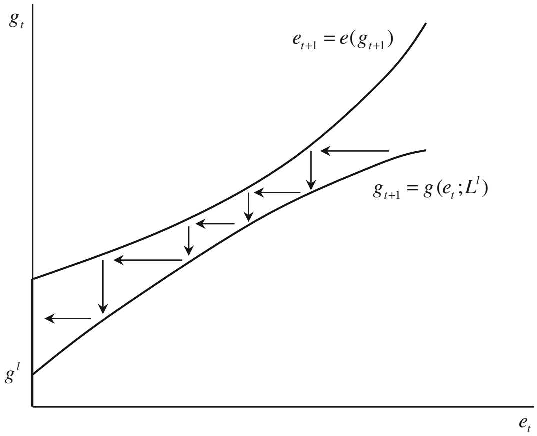

Fig. 1 Dynamics of education and technology in the early stage

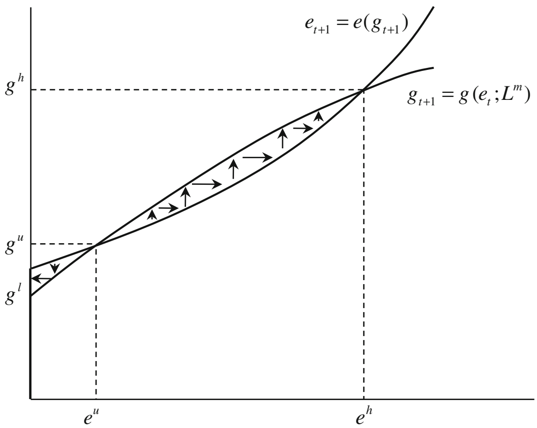

Fig. 2 Dynamics of education and technology in the intermediate stage

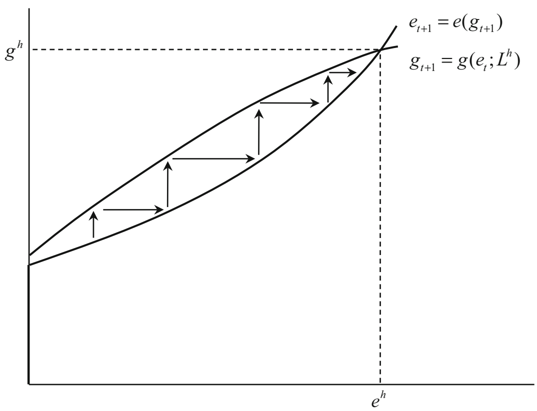

Fig. 3 Dynamics of education and technology in the advanced stage

and GW's. They are the same in terms of the EE locus. The first difference lies on the Malthusian Frontier which in GW separates the one regime where the correlation between parental income and child quantity is positive from the other where the correlation is zero. The second difference is on the shape of the XX locus because the current model generates a hump-shaped relationship between population growth and income.

The Conditional Malthusian Frontier

According to Proposition 1, the economy switches the regime from the one where individuals' income has a positive effect on their chosen number of children to the other where the effect becomes negative when potential income, z_t, exceeds its time varying critical level, $\tilde{z}_t$.

For the dynamical system in (24), the Conditional Malthusian Frontier, $MM_{|g_t}$, is the set of all pairs (e_t, x_t) conditional on given g_t and $z_t = \tilde{z}_t \equiv \tilde{z}(g_{t+1}; p)$. More formally, $MM_{|g_t}$ is written as

$$MM_{|g_t} \equiv \left\{(e_t, x_t) : x_t^{1-\alpha} h(e_t, g_t)^{\alpha} = \tilde{z}(g(e_t)) \mid g_t\right\}.$$

Lemma 1 *If $(e_t, x_t) \in MM_{|g_t}$, x_t is a monotonically decreasing function of e_t. Moreover, a decrease in x_t along $MM_{|g_t}$ is larger than that in the case where $\tilde{z}_t$ is constant for an equal increase in e_t. Furthermore, the critical $\tilde{z}_t$ decreases along $MM_{|g_t}$ as e_t increases.*

Proof See Appendix. □

The Conditional Malthusian Frontier is similar to that in GW in the sense that it is a downward sloping curve, intersects the x_t axis and approaches to the e_t axis asymptotically as x_t approaches to zero. As the functional forms are implicit, however, we cannot predict the second order property of the frontier while it is a strictly convex function in GW. An increase in g_t shifts the frontier upward as in GW. Without loss of generality, $MM_{|g_t}$ is depicted as a downward sloping convex curve in Figs. 4, 5 and 6.[24]

The frontier is affected by the evolution of population, L, and the price of the leisure good, p.

Lemma 2 *(a) An increase in L and a decrease in p lead $MM_{|g_t}$ to shift downward and leftward.*

Proof See Appendix. □

The intuition of the results in Lemma 2 is that the boundary of the Malthusian region where potential income has a positive effect on child quantity will shrink as the leisure good becomes cheaper and population size increases.[25]

The XX Locus

According to (22), the effective resources per worker, x_t, is constant if growth rates of working population and technology are equal. The conditional XX locus is the set of all pairs (e_t, x_t) for given g_t, such that x_t is in a steady state. More formally,

$$XX_{|g_t} \equiv \{(e_t, x_t) : x_{t+1} = x_t \mid g_t\}.$$

[24] The second order property of the Conditional Malthusian Frontier would be irrelevant to the qualitative analysis.

[25] By setting $p_t = \infty$ and following GW's utility maximization approach, we can derive their frontier as $\Omega_t = 0$ in (14).

Lemma 3 *There exists a unique value* $0 < \hat{e} < e^h$ *such that (a) for each* $0 \leq e_t < \hat{e}$*, there are two values for* $x_t \in XX_{|g_t}$*:* $x_t^h > x_t^l$ *such that* $z(e_t, x_t^h) > \tilde{z}_t$ *and* $z(e_t, x_t^l) < \tilde{z}_t$*, and a unique value* $x_t^l < \tilde{x}_t < x_t^h$ *such that* $z(e_t, \tilde{x}_t) = \tilde{z}_t$*; (b) for* $e_t = \hat{e}$*, there is a unique* $\hat{x} \in XX_{|g_t}$ *such that* $z(\hat{e}, \hat{x}) = \tilde{z}_t$*; and (c) for* $\hat{e} < e_t \leq e^h$*, there is no* $x_t \in XX_{|g_t}$*. Moreover, for* $z_t \geq \tilde{z}_t$*,*

$$x_{t+1} - x_t \begin{cases} > 0 & if\ [(e_t, x_t) > (e_t, x_t^h(e_t))\ and\ 0 \leq e_t < \hat{e}],\ [(\hat{e}, x_t) > (\hat{e}, \hat{x})]\ or\ [e_t > \hat{e}] \\ = 0 & if\ [(e_t, x_t) = (e_t, x_t^h(e_t))\ and\ 0 \leq e_t < \hat{e}]\ or\ [(e_t, x_t) = (\hat{e}, \hat{x})] \\ < 0 & if\ (e_t, x_t) < (e_t, x_t^h)\ and\ 0 \leq e_t < \hat{e}. \end{cases}$$

For $z_t \leq \tilde{z}_t$*,*

$$x_{t+1} - x_t \begin{cases} < 0 & if\ (e_t, x_t) > (e_t, x_t^l(e_t))\ and\ 0 \leq e_t < \hat{e} \\ = 0 & if\ [(e_t, x_t) = (e_t, x_t^l(e_t))\ and\ 0 \leq e_t < \hat{e}]\ or\ [(e_t, x_t) = (\hat{e}, \hat{x})] \\ > 0 & if\ [(e_t, x_t) < (e_t, x_t^l(e_t))\ and\ 0 \leq e_t < \hat{e}],\ [(\hat{e}, x_t) < (\hat{e}, \hat{x})]\ or\ [e_t > \hat{e}]. \end{cases}$$

Proof See Appendix. □

The locus $XX_{|g_t}$ is strictly below the curve $MM_{|g_t}$ for $e_t < \hat{e}$ and $x_t < \hat{x}$, but strictly above the curve $MM_{|g_t}$ for $e_t < \hat{e}$ and $x_t > \hat{x}$. At $(\hat{e}, \hat{x})$, the curve $MM_{|g_t}$ and the locus $XX_{|g_t}$ coincide. Since the curve $n(x_t, e_t; L, p)$ shifts downward as e_t increases, x_t^l increases while x_t^h decreases unambiguously. Hence the part of the locus $XX_{|g_t}$ below (above) the curve $MM_{|g_t}$ is depicted in Figs. 4, 5 and 6 as an upward (downward) sloping curve.[26]

The locus $XX_{|g_t}$ is also affected by the evolution of population and the price of the leisure good.

Lemma 4 *An increase in* L *and a decrease in* p *lead* $XX_{|g_t}$ *under* $MM_{|g_t}$ *to shift up, but above* $MM_{|g_t}$ *to shift down. Furthermore, the critical* $\hat{e}$ *decreases as* L *increases and* p *decreases.*

Proof See Appendix. □

An interesting intuition of the result in Lemma 4 is that the economy may cross over the Malthusian Frontier into the Modern Growth region as the price of the leisure good decreases for a given level of education.[27]

The EE locus

The conditional EE locus is exactly the same as that in GW, that is a set of all pairs (e_t, x_t) conditional on given g_t such that education per worker e_t is in a steady state:

$$EE_{|g_t} \equiv \{(e_t, x_t) : e_{t+1} = e_t \mid g_t\}.$$

GW shows that the steady-state values of e_t are independent of g_t and x_t, for a given size of population. Therefore the locus EE is a vertical line in the (e_t, x_t) space and shifts rightward as population size increases. The location of the locus EE identifies one of three phases of

26 GW assume that the total derivative $\partial z(h_t, x_t)/\partial g_t > 0$ (holding A_{t-1} constant) although the partial derivative is negative (holding x_t and thus A_t constant). This assumption is maintained in the current model.

27 The locus $XX_{|g_t}$ in GW is vertical at $\hat{e}$ above the Malthusian Frontier which can be derived by setting $p_t = \infty$ as well as following their utility maximization approach. Under such circumstances, one must maintain the assumption (A4) in GW to ensure that the XX locus is nonempty for $z_t \geq \tilde{z}$, that is, $\hat{g} < (\gamma/\tau^q) - 1 < g(e^h(L_0), L_0)$.

economic development in terms of the evolution of education and technology. In the early stage of development, the locus EE is vertical at $e = 0$ representing the globally stable temporary steady-state equilibrium, $(\bar{e}, \bar{g}) = (0, g^l)$ and

$$e_{t+1} - e_t \begin{cases} = 0 & \text{if } e_t = 0 \\ < 0 & \text{if } e_t > 0. \end{cases} \tag{25}$$

The intermediate stage of development is characterized by the multiple locally stable temporary steady-state equilibria, $(0, g^l)$, (e^u, g^u) and (e^h, g^h) so that the locus EE is vertical at $e_t = 0$, $e_t = e^u$ and $e_t = e^h$. The ones at $e_t = e^u$ and $e_t = e^h$ shift rightward as population size increases. The global dynamics of e_t are given by

$$e_{t+1} - e_t \begin{cases} = 0 & \text{if } e_t \in \{0, e^u, e^h\} \\ > 0 & \text{if } e^u < e_t < e^h \\ < 0 & \text{if } 0 < e_t < e^u \text{ or } e_t > e^h. \end{cases} \tag{26}$$

In the advanced stage of development, the locus EE at $e_t = e^h$ represents a globally stable steady-state equilibrium, (e^h, g^h). It shifts rightward as population size increases. The global dynamics of e_t in this case is given by

$$e_{t+1} - e_t \begin{cases} = 0 & \text{if } e_t = e^h \\ > 0 & \text{if } 0 \le e_t < e^h \\ < 0 & \text{if } e_t > e^h. \end{cases} \tag{27}$$

4.2 Conditional steady-state equilibria

The dynamical system in the early stage of economic development with small population sizes is characterized by two conditional steady-state equilibria which are given by the intersection between the XX locus and the EE locus in the (e_t, x_t) space, as shown in Fig. 4.

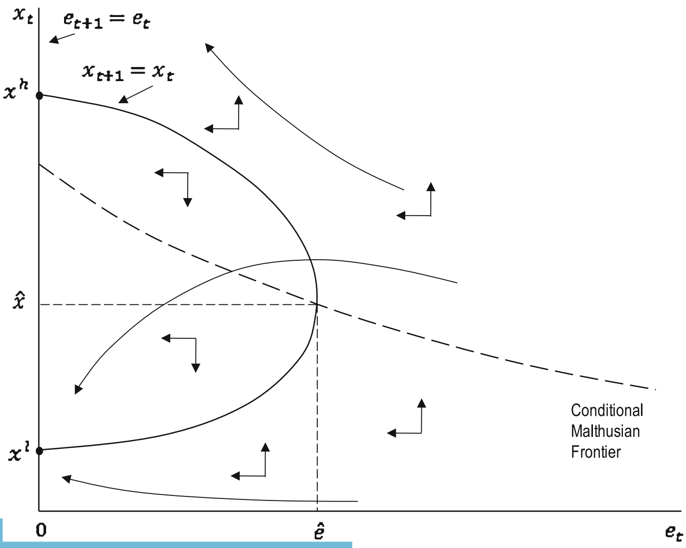

Fig. 4 The conditional dynamical system in the early stage

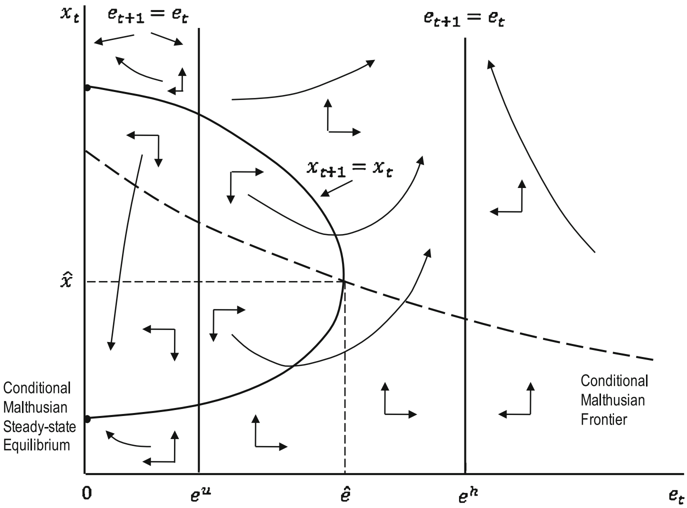

Fig. 5 The conditional dynamical system in the intermediate stage

Both equilibria are conditional on the rate of technological progress, the size of population and the price of the leisure good. Since the conditional steady-state equilibrium $(\bar{e}, \bar{x}) = (0, x^h)$ is unstable, the locally stable conditional steady-state equilibrium $(\bar{e}, \bar{x}) = (0, x^l)$ is the Malthusian steady-state. Another reason why the unstable steady-state equilibrium is not the Malthusian steady-state is that an increase in potential income has a negative effect on child quantity which is opposite to the assumption on which the Malthusian model is built.

The dynamical system in the intermediate stage of development with moderate population sizes, depicted in Fig. 5, is similar to that in GW in the sense that the Malthusian conditional steady-state is locally stable and the conditional steady-state equilibrium $(e^u, x^l(e^u))$ is a saddlepoint. In addition to those in GW, there are two conditional unstable steady-state equilibria: $(0, x^h(0))$ and $(e^u, x^h(e^u))$. If the level of education is above e^u, the dynamical system converges to an equilibrium with a level of education e^h and possibly a steady-state growth rate of x_t, given the population size and the price of the leisure good. In the advanced stage of development with large population sizes, the dynamical system is, as depicted in Fig. 6, converges globally to an education level e^h and possibly a steady-state growth rate of x_t, given the population size and the price of the leisure good.

4.3 Analysis

Collecting the results obtained so far, we can now discuss the transition of economic development from the Malthusian regime to the Modern Growth regime. Consider that the economy is in the early stage of development where population size is sufficiently small, the rate of technological progress is so low in the consumption good sector that parents find it inefficient to invest in their children's education. In addition, the price of the leisure good is sufficiently high and constant as the technology of producing this good is primitive and stagnant so that its consumption/production is small. In other words, children are much cheaper than the leisure good. In Fig. 4, the situation is represented by the temporary, conditional and locally stable Malthusian steady-state equilibrium where both the level of education and effective resources per worker are constant, for a given rate of technological change in the consumption good

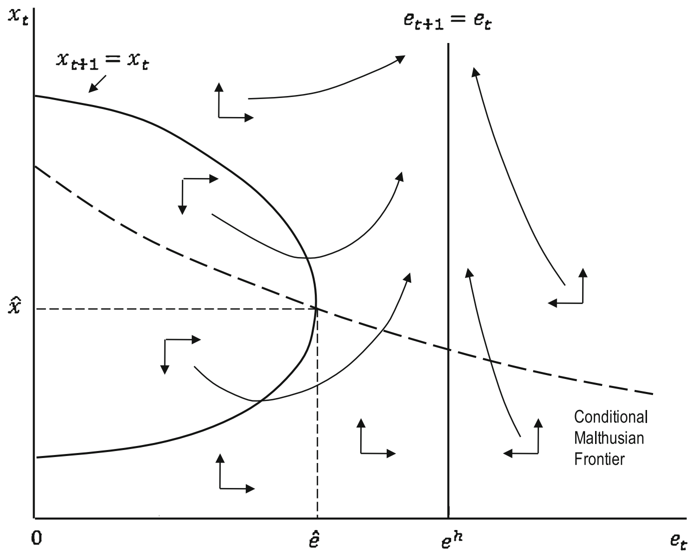

Fig. 6 The conditional dynamical system in the advanced stage

sector. Consequently, output per capita is constant from (1) and (13). Thus population grows slowly, at the same rate as technological progress, g_{t+1}. At an extremely high price of the leisure good, the unstable conditional steady-state equilibrium, $(0, x^h)$, may not exist so that the Malthusian steady-state equilibrium can be globally stable as the XX locus for $z_t > \tilde{z}_t$ (i.e., above the Malthusian Frontier) may become vertical at $\hat{e}$ as in GW. Temporary shocks to technology and population will be adjusted towards the Malthusian steady-state.

Over time, the price of the leisure good may decrease as population reaches a sufficiently large size and creates the unstable conditional steady-state equilibrium (i.e., the XX locus for $z_t > \tilde{z}_t$ is no longer vertical at $\hat{e}$). If the effective resources per worker jump above the XX locus for $z_t > \tilde{z}_t$ due to temporary shocks to population and technology, the system will converge to zero education level and possibly a steady-state growth rate of x_t, given the population size, the rate of technological progress and the price of the leisure good. Under such circumstances, the number of children per person will decrease until the minimum quantity of children constraint binds as children are more expensive relative to the leisure and consumption goods in this region. In other words, it will generate a demographic transition. However, we do not consider this possibility. Instead, we assume that the economy stays around the Malthusian steady-state at this early stage of development.

When the size of the population reaches a sufficiently high level, the dynamical system will be characterized by multiple steady-state equilibria as in GW: the Malthusian steady-state with constant income per capita, slow technological progress and zero education, and the Modern Growth steady-state with fast technological progress, a high level of education and increasing income per capita. As depicted in Fig. 5, the convergence towards these steady states is history dependent. In addition, the economy may jump over the boundary and evolve accordingly due to shocks to technology and education. Like GW, however, we are interested in the economy starting out in the Malthusian steady-state stays there at this intermediate stage.

Figure 3 shows that the evolution of education and technology is monotonic and converges to a unique globally stable steady-state with fast technological progress and a high level of education when the size of population reaches a sufficiently high level. Simultaneously, the Malthusian steady-state disappears. As in GW, technological progress has opposite effects on the evolution of population growth in the Malthusian region of Fig. 6. As shown in Proposition 1, technological progress has a negative effect on the number of children per adult through an increase in education per child and an overall decrease in human capital per child, for a given level of potential income and a given price of the leisure good. At the same time, it increases parental potential income which has a positive effect on the number of children per person. If the technology of producing the leisure good advances, child quantity will tend to decrease as the leisure good becomes cheaper. Initially, the positive effect dominates all the negative effects hence the rate of population growth will increase, reflecting the characteristics of the Post-Malthusian regime.

If the positive effect of technological change continues to dominate the negative effects on the number of children per person, the rate of population growth will increase continuously until the economy crosses over the Malthusian Frontier. As soon as the economy enters the Modern Growth region, the rate of population growth will decrease unambiguously as the growth in parental potential income produces a negative effect on the number of children per person. Income per capita continues to increase while the rate of population growth continues to decrease even without any further improvements in technology (i.e., education per person, human capital per person and the price of the leisure good are all constant). The source of growth for income per capita will be the growth of effective resources per capita as the rate of technological change is faster than population growth in this region. This is one of the main differences between the current analysis and that in GW. Under the same circumstances, GW would predict a constant growth rate of population but growing income per capita as parental potential income has no effect on fertility. Only faster technological changes lead to a decrease in population growth by raising education. In the current model, in contrast, an increase in education lead to an even faster decline in population growth as it complements with the negative effect of growing income per capita. A decrease in the price of the leisure good due to technological progress speeds up the transition into the Modern Growth region by shifting the Malthusian Frontier down (i.e., decreasing the critical level of education). Moreover, it leads to a faster decrease in fertility alongside an increase in income and an increase in education once the economy leaves the Malthusian region behind.

The decrease in population growth is bounded from below as the minimum quantity of children constraint binds. If the minimum quantity of children is one per adult, population growth will be zero in the Modern Growth regime. Under such circumstances, the economy converges to a global steady state in which both education and the rate of technological progress are constant as population size is constant. The leisure good, on the other hand, may be falling in relative price terms. If the minimum number of children is above (below) one, the size of population will increase (decrease) and hence the evolution of education and technology will change accordingly.

5 Conclusion

The paper analyzes the role of the rise in the cost of children relative to leisure goods in the decline in fertility observed across the world during the course of modern economic history. To explain this mechanism, we have extended the unified growth model of GW by generalizing their utility function in which consumption and children are independent of each other, with one in which children and leisure goods are substitutes for leisure activities while consumption is unrelated with those leisure activities. Consequently, the demographic transition from high to low fertility is the outcome of three events in this model, rather than one in GW. For a given level of education and a given price of leisure goods, the fertility decline is a natural phenomenon when children becomes relatively more expensive than leisure goods in enhancing parental welfare, at high income levels. An increase in educational attainment (which is the mechanism in GW) and a decrease in the price of leisure goods lead to a faster fertility transition by making child quantity even more expensive from parents' point of view.

Appendix

Proof of Proposition 1 The results in parts (a) and (b) follow directly from differentiating n_t in (14), d_t in (15), l_t in (16) and u_t in (17) with respect to p_t and g_{t+1} after substituting (13) and (6) into (14) and (15). The signs of $n_z(\cdot)$ and $l_z(\cdot)$ in part (c) are determined by:

$$sgn\left\{n_z(\cdot) = l_z(\cdot)\right\} \equiv sgn\left\{\tilde{c}(1-\sigma) - \Omega_t(\nu\sigma z_t - \tilde{c}(1-\sigma(1-\nu))\right\}.$$

It is clear that $n_z(\cdot) = l_z(\cdot) > 0$ for $z_t \leq \frac{\tilde{c}(1-\sigma(1-\nu))}{\sigma\nu}$, but $n_z(\cdot) = l_z(\cdot) \gtreqless 0$ for $z_t > \frac{\tilde{c}(1-\sigma(1-\nu))}{\sigma\nu}$. If $\sigma = 0$, it is true that $n_z(\cdot) = l_z(\cdot) > 0$. If $\sigma = 1$, the reverse is true. In an ideal case with $\sigma \in (0, 1)$, the second term, $\Omega_t(\nu\sigma z_t - \tilde{c}(1-\sigma(1-\nu))$, is a monotonically increasing function of z_t. Since $\tilde{c}(1-\sigma)$ is constant, there exists a unique $\tilde{z}_t \equiv \tilde{z}(p_t, g_{t+1})$ such that $n_z(z_t = \tilde{z}_t) = l_z(z_t = \tilde{z}_t) = 0$ or $\tilde{c}(1-\sigma) = \Omega_t(\nu\sigma z_t - \tilde{c}(1-\sigma(1-\nu))$. Given (13), (6) and $h_g(g_{t+1}) < 0$, the implicit function theorem suggests that $\tilde{z}_g(\cdot) < 0$ and $\tilde{z}_p(\cdot) > 0$.

Proof of Lemma 1 Given the result in part (c) of Proposition 1 that $\tilde{z}_t$ is a decreasing function of g_{t+1}, an increase in e_t has a negative effect on $\tilde{z}_t$ through an increase in g_{t+1}. Since h_t is an increasing function of e_t, x_t must decrease in response to an increase in e_t along $MM_{|g_t}$.

Proof of Lemma 2 According to (18), an increase in L leads to an increase in g_{t+1}. Given the result in part (c) of Proposition 1, both an increase g_{t+1} and a decrease in p have a negative effect on $\tilde{z}_t$ so that x_t must decrease for given e_t.

Proof of Lemma 3 First rewrite (14) as follows:

$$n_t = \frac{\gamma\beta\left(1 - \tilde{c}/z_t\right)}{(\tau^q + \tau^e e_{t+1})\left(1 + \Psi_t z_t^{\frac{\nu\sigma}{1-\sigma}}\right)} \equiv n(e_t, x_t) \tag{28}$$

where $\Omega_t = \Psi_t z_t^{\frac{\nu\sigma}{1-\sigma}}$, $\Psi_t = M\left(\frac{(\tau^q+\tau^e e_{t+1})^\beta}{h_{t+1}^\beta p_t^\nu}\right)^{\frac{\sigma}{1-\sigma}}$, $e_{t+1} = e(g_{t+1})$, $h_{t+1} = h(g_{t+1})$, $g_{t+1} = g(e_t; L)$ and $z_t = x_t^{1-\alpha}h(e_t, g_t)^\alpha$ for given L and p. We find that $n_x(e_t, x_t) = \tilde{c}(1-\sigma) - F(x_t, e_t)$ where $F(x_t, e_t) = \Psi_t z_t^{\frac{\nu\sigma}{1-\sigma}}(\sigma\nu z_t - (1-(1-\nu)\sigma)\tilde{c})$ which is monotonically increasing in x_t for each e_t—i.e., $F_x(x_t, e_t) > 0$.[28] Since $\tilde{c}(1-\sigma)$ is independent of x_t, there exists a unique $\tilde{x}_t$ for given e_t, such that $\tilde{c}(1-\sigma) = F(x_t, e_t)$ and $\tilde{z}_t = \tilde{x}_t^{1-\alpha}h(e_t, g_t)^\alpha$—i.e., the pair $(\tilde{x}_t, e_t)$ is on $MM_{|g_t}$. Both $\tilde{c}(1-\sigma)$ and $F(x_t, e_t)$ are depicted in Fig. 7a where $\ddot{x}(e_t)$ is determined from $F(x_t, e_t) = 0$.[29] For $x_t < \tilde{x}_t$, $\tilde{c}(1-\sigma) > F(x_t, e_t)$ hence $n_x(x_t, e_t) > 0$.

[28] This is essentially the same expression found in Proof of Proposition 1.

[29] The second order property of $F(x_t, e_t)$ is ambiguous depending the values of α and σ. However, the qualitative analysis is not affected by this. Hence without loss of generality, $F(x_t, e_t)$ is depicted as a straight line in Fig. 7.

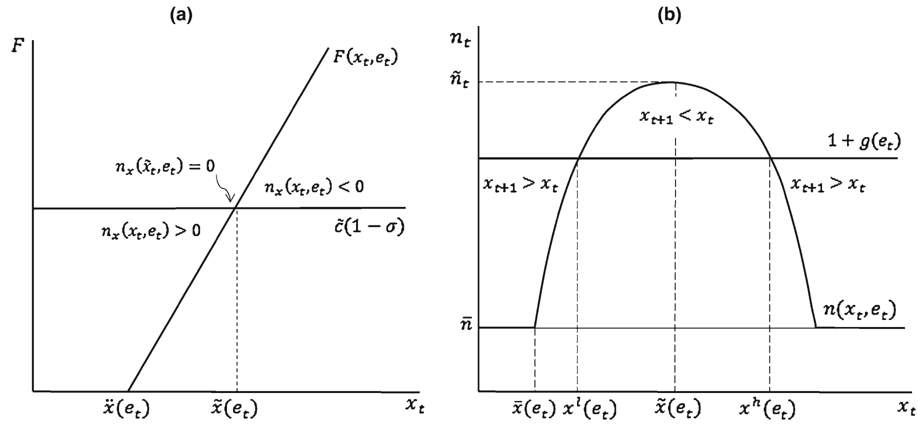

Fig. 7 Derivation of the XX locus

For $x_t > \tilde{x}_t, \tilde{c}(1-\sigma) < F(x_t, e_t)$ hence $n_x(x_t, e_t) < 0$. Thus there is a hump-shaped relationship between n_t and x_t for each e_t. Hence we can obtain $\tilde{n}_t = n(\tilde{x}_t, e_t)$ such that $n_x(\tilde{x}_t, e_t) = 0$.

Now depict both $1 + g(e_t)$ and $n(x_t, e_t)$ in the (n_t, x_t) space which is given in Fig. 7b where $\bar{x}(e_t)$ can be determined from (14) such that $n_t = \bar{n}$. Since $1 + g(e_t)$ is independent of x_t, it is a horizontal line. The intersection of $1 + g(e_t)$ and $n(x_t, e_t)$ yields two steady-state values, $x^l(e_t)$ and $x^h(e_t)$, such that $x^l(e_t) < \tilde{x}(e_t) < x^h(e_t)$ if $\tilde{n}_t > 1 + g(e_t)$. Furthermore, $x_{t+1} - x_t > 0$ for both $x_t < x^l(e_t)$ and $x_t > x^h(e_t)$ as $n_t < 1 + g(e_t) < \tilde{n}_t$. However, $x_{t+1} - x_t < 0$ for $x^l(e_t) < x_t < x^h(e_t)$ as $1 + g(e_t) < n_t \leq \tilde{n}_t$.

Let us now analyze the situation where e_t increases. For each x_t, $F(x_t, e_t)$ increases as both z_t and Ψ_t increase. For the new e_t and Ψ_t, a decrease in $\tilde{x}_t$ must be sufficient to restore $\tilde{c}(1-\sigma) = \Psi_t \tilde{z}_t^{\frac{\sigma}{1-\sigma}}(\sigma \nu \tilde{z}_t - (1-(1-\nu)\sigma)\tilde{c})$. In particular, the new $\tilde{x}_t$ must be smaller than before to ensure that the new $\tilde{z}_t$ is smaller than before. Thus $\tilde{n}_t$ is smaller than before. It implies that the point, $(\tilde{n}_t, \tilde{x}_t)$, shifts leftward and downward in the (n_t, x_t) space. In other words, $\tilde{n}_t$ decreases monotonically as e_t increases. Since $1 + g(e_t)$ increases as e_t increases, there exists a unique $\hat{e}$ such that $\tilde{n}_t = 1 + g(\hat{e})$. For $\tilde{n}_t = 1 + g(\hat{e})$, $\tilde{x}_t = \hat{x}$ and the pair $(\hat{e}, \hat{x})$ is on $MM_{|g_t}$. Since $\tilde{n}_t < 1 + g(e_t)$ for $\hat{e} < e_t \leq e^h$, there is no $x_t \in XX_{|g_t}$.

Proof of Lemma 4 An increase in L leads to $\phi(\cdot) = (1 + g_{t+1})/n_t > 1$ through its direct positive effect on g_{t+1} and an indirect negative effect on n_t which works through an increase in e_{t+1}, a decrease h_{t+1} and hence an increase in Ψ_t. Thus n_t must increase to restore the steady-state condition for x_t, $\phi(\cdot) = 1$. It must be achieved as x_t^l increases (i.e., z_t increases) and x_t^h decreases (i.e., z_t decreases) for each e_t. The increase in Ψ_t leads to a decrease in the corresponding $\tilde{x}_t$ by shifting $F(x_t, e_t)$ leftward which is consistent with the result in Lemma 5. For given e_t, a decrease in $\tilde{x}_t$ leads to a decrease in $\tilde{z}_t$ so that $\tilde{n}_t$ decreases. Thus $\hat{e}$ decreases. A decrease in p has the same effect which works through a decrease in n_t for $\phi(\cdot)$.

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